LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **PHYSICS**

THIRD SEMESTER – **NOVEMBER 2018**

16/17UPH3MC01– MATHEMATICAL PHYSICS

Date: 29-10-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART - A

ANSWER **ALL** QUESTIONS **(10 × 2 = 20 MARKS)**

1. Express in polar form: $1 - \sqrt{2} + i$
2. Prove that $(\cosh x - \sinh x)^n = \cosh nx - \sinh nx$.
3. Find the directional derivative of $\phi(x, y, z) = x^2yz + 4xz^2$ at (1, -2, 1) in the direction of

$$2\hat{\imath} - \hat{\jmath} - 2\hat{k}.$$

4. Define scalar and vector point functions.
5. What are periodic functions? Give examples.
6. Find the Fourier transform of

$$f(x) = \begin{cases} 1 & for \ \ |x| < a \\ 0 & for \ \ |x| > a \end{cases}$$

7. What is the basic difference between the solution of one dimensional wave equation and one dimensional heat equation?
8. Express Laplacian in spherical coordinates.
9. What are the advantages of Lagrange's formula over Newton's formula?
10. Using Euler's method find y(0.2) and y(0.4) from $\frac{dy}{dx} = x + y$, y(0) = 1 with h = 0.2.

PART - B

ANSWER ANY **FOUR** QUESTIONS **(4 × 7.5 = 30 MARKS)**

11. Define a harmonic function. Show that the function $u(x, y) = x^4 - 6x^2y^2 + y^4$ is harmonics. Also find the analytic function $f(z) = u(x, y) + iv(x, y)$.
12. Find the line integral value of $\int_c (x + y)dx + x^2\,ydy$ along i) $y = x^2$ having (0,0) and (3,9) as end points ii) along $y = 3x$ between the same points **(3.5+4)**
13. Find the Fourier sine integral for $f(x) = e^{-\beta x}$ and hence show that

$$\frac{\pi}{2}e^{-\beta x} = \int_0^\infty \frac{\lambda \sin\lambda x}{\beta^2 + \lambda^2} d\lambda$$

14. Derive the one-dimensional wave equation of a vibrating string.

15. Evaluate $\int_{-3}^{3} x^4\,dx$ by using Trapezoidal rule and Simpson's one third rule. Verify your answer with actual integration.

16. Using Newton's forward interpolation formula, find f(1.5) from the following data.

x	0	1	2	3	4
f(x)	858.3	869.6	880.9	829.3	903.6

PART - C

ANSWER ANY **FOUR** QUESTIONS **(4 × 12.5 = 50 MARKS)**

17. (i)Derive Cauchy-Riemann equations for a function to be analytic

    (ii)Evaluate the following integral using Cauchy integral formula

    $\int_c \frac{4-3z}{z(z-1)}dz$ where c is the circle $|z| = \frac{3}{2}$**(8+4.5)**

18. i) If $u - v = (x - y)(x^2 + 4xy + y^2)$, then find the corresponding analytic function $f(z) = u + iv$ (8.5)

    ii) Write $\sin\,(x + iy)\ and\ \cos\,(x + iy)$ into its real and imaginary parts. **(4)**

19. (i) Prove that $(y^2 - z^2 + 3yz - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational. **(3+3)**

    (ii)Verify Green's theorem in the plane for $\oint_c (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x^2$and $y = x$. (6.5)

20. A sinusoidal voltage $E\sin\omega t$, where t is time is passed through a half-wave rectifier that clips the negative portion of the wave. Find the Fourier series of the resulting periodic function

$$u(t) = \begin{cases} 0 & if \quad -L < t < 0, \\ E\sin\omega t & if \quad 0 < t < L \end{cases} \qquad p = 2L = \frac{2\pi}{\omega}, \quad L = \frac{\pi}{\omega}$$

21. Derive the solution of wave equation by D'Alembert's method.

22. (i) Find the positive root of $x^4 - x = 10$ and correct to three decimal places using Newton's Raphson method.

    (ii) Use Lagrange's formula to calculate f(3) from the following table.

x	0	1	2	4	5	6
f(x)	1	14	15	5	6	19

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